COMPARING PERFORMANCE OF SIMPLE ADDITIVE WEIGHTING AND WEIGHTED PRODUCT METHODS FOR SPHERICAL FUZZY DECISION-MAKING

Nik Muhammad Farhan Hakim Nik Badrul Alam[1*], Nazirah Ramli[2]

[1, 2]*Faculty of Computer and Mathematical Sciences*
Universiti Teknologi MARA UiTM Pahang, 26400 Bandar Tun Abdul Razak Jengka, Pahang Malaysia

[*]*Corresponding author: farhanhakim@uitm.edu.my*

Abstract

The nature of decision-making is commonly fashioned by uncertainty and vagueness due to lack of information and psychological biases of human beings. Many efforts have been made to cater this issue such the emergence of fuzzy set theory. Recently, the fuzzy set theory has been extended to spherical fuzzy sets (SFS) which generalize other forms of extension of fuzzy sets. The SFS consists of membership, non-membership and hesitancy grades, which are independent of each other, providing larger domains for decision-makers to assign such grades. SFS makes use of the three variables in its calculation as compared to only membership grade in the classical fuzzy set. In this paper, the performance of the SFS using simple additive weighting (SAW) and weighted product method (WPM) for ranking alternatives was compared. The developed models were illustrated using coffee shop selection in Jengka, Pahang based on six criteria: atmosphere, flavor, price, menu variety, service and location. The SAW and WPM models based on spherical fuzzy numbers were applied and compared in ranking three coffee shops. Further, sensitivity analysis was performed to validate the consistency and stability of the models by increasing and decreasing the criteria weights by 50%. As a result, the developed models show unchanged ranking of alternatives (coffee shops) which highlights the stability of the models. Comparing the two models, the WPM model performs better than the SAW model since it produces smaller errors in terms of score functions when compared to the initial values.

Keyword: Coffee shop selection, simple additive weighting, spherical fuzzy set, weighted product method

Introduction

The information is sometimes unclear during the decision-making process, which leads to uncertainty and vagueness. The use of crisp numbers in describing certain variables is less suitable as humans tend to exhibit hesitation due to lack of information and psychological biases (Aliev et al., 2021). Hence, the emergence of fuzzy set theory (Zadeh, 1965) helps to overcome this issue by allowing certain values in the interval [0,1] to describe a variable instead of using only 0 and 1 to represent the falsity and truth.

Fuzzy set theory itself has shown extension into intuitionistic fuzzy set (IFS), Pythagorean fuzzy set and picture fuzzy set. Recently, Kutlu Gündoğdu and Kahraman (2019) introduced the concept of spherical fuzzy set (SFS) which generalizes other forms of extensions of fuzzy sets. The SFS consists of membership, non-membership and hesitancy grades, which

are independent of each other. This feature allows the decision-makers to express their judgement efficiently (Sharaf, 2021). For instance, the linear restriction of the membership and non-membership in the IFS leads to triangular preference domain. However, the restriction of the membership, non-membership and hesitancy grades in the SFS forms a spherical surface, which provides flexibility in expressing the preferences by decision-makers.
The SFS has been applied in various fuzzy decision-making approaches such as simple additive weighting (SAW) (Kutlu Gündoğdu & Yörükoğlu, 2021),TOPSIS (Aydoğdu et al., 2023), ARAS (Aydoğdu & Gül, 2022), PROMETHEE (Akram et al., 2023), WASPAS (Wang et al., 2022) and EDAS (Garg & Sharaf, 2022). Each of these approaches has its own strengths and weaknesses. The simple additive weighting (SAW) and weighted product method (WPM) exhibit the simplicity in evaluating the criteria weights and ranking the alternatives.

Fauzi et al. (2021) adopted the analytic hierarchy process (AHP) to determine the most preferred locations for coffee shop business by considering seven criteria which were rental rate, buildings, water supply, internet network, electricity supply, accessibility, and distances from offices. On the other hand, Purba et al. (2023) used TOPSIS method to determine the best coffee shop by taking into account five attributes which were price, service, flavor, facility and location. However, the crisp value was used to develop the TOPSIS method, which was incapable of handling uncertainty of decision-making.

Furthermore, Hutagalung et al. (2023) developed VIKOR method to assess customers' preferences of coffee shops in Medan, Indonesia. In their approach, the atmosphere, flavor, menu variety, price, facility and service were among the criteria in determining the customers' preferences for a certain coffee shop. Again, the crisp valuation using the proposed VIKOR method was limited to uncertainty and vagueness among respondents. Recently, Windarto et al. (2024) adopted the preference selection index (PSI) approach to determine the best coffee shop considering five criteria, namely the food, beverages, service, entertainment and parking area.

Hence, this paper adopts the simple additive weighting (SAW) and weighted product method (WPM) from Kutlu Gündoğdu and Yörükoğlu (2021) to rank coffee shops based on six criteria: atmosphere, flavor, price, menu variety, service and location. The two fuzzy decision-making methods used in this paper perform the calculations using spherical fuzzy numbers (SFN).

The organization of this paper is summarized as follows: this section provides some background on the topic; Section 2 reviews some preliminaries related to the SFN; Section 3 describes the proposed spherical SAW and WPM models; Section 4 illustrates the developed decision-making models for coffee shop selection; Section 5 discusses the obtained results and Section 6 concludes the paper.

Preliminaries

In this section, some basic concepts on the spherical fuzzy numbers (SFN) are reviewed as follows:

Definition 1 (Kutlu Gündoğdu & Kahraman, 2019)
A spherical fuzzy set in the universe of discourse U is represented by

$$A = \left\{ x, \left(\mu_A(x), \nu_A(x), \pi_A(x) \right) \mid x \in U \right\} \quad (1)$$

where $\mu_A(x): U \to [0,1]$, $\nu_A(x): U \to [0,1]$, $\pi_A(x): U \to [0,1]$, and

$$0 \le \mu_A^2(x) + \nu_A^2(x) + \pi_A^2(x) \le 1 \quad \forall x \in U. \quad (2)$$

Next, some basic operations on the SFN are reviewed.

Definition 2 (Kutlu Gündoğdu & Kahraman, 2019)
Let $A=(\mu_A,\nu_A,\pi_A)$ and $B=(\mu_B,\nu_B,\pi_B)$ be two SFN and λ be a non-negative scalar. Then

(i) $A\oplus B=\left(\sqrt{\mu_A^2+\mu_B^2-\mu_A^2\mu_B^2},\nu_A\nu_B,\sqrt{(1-\mu_B^2)\pi_A^2+(1-\mu_A^2)\pi_B^2-\pi_A^2\pi_B^2}\right)$ (Addition)

(ii) $A\otimes B=\left(\mu_A\mu_B,\sqrt{\nu_A^2+\nu_B^2-\nu_A^2\nu_B^2},\sqrt{(1-\nu_B^2)\pi_A^2+(1-\nu_A^2)\pi_B^2-\pi_A^2\pi_B^2}\right)$ (Multiplication)

(iii) $\lambda\cdot A=\left(\sqrt{1-(1-\mu_A^2)^\lambda},\nu_A^\lambda,\sqrt{(1-\mu_A^2)^\lambda-(1-\mu_A^2-\pi_A^2)^\lambda}\right)$ (Scalar multiplication)

(iv) $A^\lambda=\left(\mu_A^\lambda,\sqrt{1-(1-\nu_A^2)^\lambda},\sqrt{(1-\nu_A^2)^\lambda-(1-\nu_A^2-\pi_A^2)^\lambda}\right)$ (Exponentiation).

Sometimes, there are more than one decision-maker involved in the decision-making process. Hence, the aggregation of the decision-makers can be done using spherical weighted arithmetic mean (SWAM) which is defined below.

Definition 3 (Kutlu Gündoğdu & Yörükoğlu, 2021)
The spherical weighted arithmetic mean (SWAM) with respect to $\omega=(w_1,w_2,...,w_n)$, where $w_1+w_2+...+w_n=1$, is defined by

$$SWAM(A_1,...,A_n)=w_1A_1+w_2A_2+...+w_nA_n$$
$$=\left(\sqrt{1-\prod_{i=1}^{n}(1-\mu_{A_i}^2)^{w_i}},\prod_{i=1}^{n}\nu_{A_i}^{w_i},\sqrt{\prod_{i=1}^{n}(1-\mu_{A_i}^2)^{w_i}-\prod_{i=1}^{n}(1-\mu_{A_i}^2-\pi_{A_i}^2)^{w_i}}\right). \quad (3)$$

Converting the SFN into crisp values requires an approach called the defuzzification. In this paper, the score function will be used to quantify the SFN into crisp values.

Definition 4 (Kutlu Gündoğdu & Yörükoğlu, 2021)
Let $A=(\mu_A,\nu_A,\pi_A)$. The score function of A is given by

$$S(A)=(3\mu_A-0.5\pi_A)^2-(\nu_A-0.5\pi_A)^2. \quad (4)$$

Proposed Fuzzy Decision-Making Model

The fuzzy decision-making model developed in this paper consists of two parts: the criteria evaluation and ranking of alternatives. The criteria are evaluated using simple additive weighting (SAW), meanwhile the alternatives are further ranked using weighted product method (WPM). The detailed steps are as follows:

Step 1: Obtain the decision-makers' preferences on each criterion and convert them into SFN in accordance with Table 1.

Table 1 Linguistic terms and their corresponding spherical fuzzy numbers (Kutlu Gündoğdu & Yörükoğlu, 2021)

Linguistic Terms	Spherical fuzzy numbers
Extremely important / Extremely good	(0.9,0.1,0.1)
Very strongly important / Very strongly good	(0.8,0.2,0.2)
Strongly important / Strongly good	(0.7,0.3,0.3)
Moderately important / Moderately good	(0.6,0.4,0.4)
Fair	(0.5,0.5,0.5)
Moderately not important / Moderately bad	(0.4,0.6,0.4)
Strongly not important / Strongly bad	(0.3,0.7,0.3)
Very strongly not important / Very strongly bad	(0.2,0.8,0.2)
Extremely not important / Extremely bad	(0.1,0.9,0.1)

Step 2: Aggregate the opinions using the SWAM operator defined in Eq. (3).

Step 3: Defuzzify the SFN representing each criterion using the score function defined in Eq. (4) to obtain the weight.

Step 4: Normalize the weight of each criterion using the formula in the following.

$$\varpi_{A_i} = \frac{S(A_i)}{S(A_1)+S(A_2)+...+S(A_n)} \tag{5}$$

where ϖ_{A_i} is the normalized weight of criterion A_i while $S(A_i)$ is the weight of criterion A_i defuzzified using the score function.

Step 5: Construct the decision matrices on *m* alternatives with respect to *n* criteria and convert them into SFN by referring to Table 1.

Step 6: Aggregate the decision matrices from all decision-makers using the SWAM operator defined in Eq. (3).

Step 7: Compute the spherical weight of each alternative using simple additive weighting (SAW) and product weighting method (WPM) defined in Eq. (6) and (7), respectively.

$$SAW(Alt_i) = \sum_{j=1}^{n} (\mu_{ij}, \nu_{ij}, \pi_{ij}) \cdot \varpi_{A_j} \quad \forall i = 1,...,m \tag{6}$$

$$WPM(Alt_i) = \prod_{j=1}^{n} (\mu_{ij}, \nu_{ij}, \pi_{ij})^{\varpi_{A_j}} \quad \forall i = 1,...,m \tag{7}$$

where Alt_i is *i*-th alternative, $(\mu_{ij}, \nu_{ij}, \pi_{ij})$ is the aggregated SFN representing the evaluation on the *i*-th alternative with respect to *j*-th criterion, and ϖ_{A_j} is the normalized weight of criterion A_j.

Step 8: Defuzzify the spherical weights of the alternatives using Eq. (4) and rank the alternatives from the highest value to the lowest value.

Illustrative Example

For illustrating the developed fuzzy decision-making model, the coffee shop selection in Jengka, Pahang is considered as a case study. This study involved 5 decision-makers, with different levels of coffee enthusiasm. The decision-makers were required to rate their level of enthusiasm using linguistic terms "extremely unlike", "very strongly unlike", "strongly unlike", "moderately unlike", "fair", "moderately like", "strongly like", "very strongly like", and "extremely like", as shown in Table 2. Note that for quantifying each linguistic term in Table 2, the variables "extremely unlike" to "extremely like" are assigned values from 1 to 9, respectively. Hence, the weightage of each decision-maker is obtained by finding its ratio to the total crisp values. For example, DM1 expressed "extremely unlike", in which the crisp value is 1, which is then divided by total values of 29, resulting in 0.034 as his/her weight.

Table 2 Level of enthusiasm towards coffee of decision-makers

	DM1	DM2	DM3	DM4	DM5	Total
Linguistic terms	Extremely unlike	Very strongly like	Moderately like	Extremely like	Fair	-
Crisp value	1	8	6	9	5	29
Weights	0.034	0.276	0.207	0.301	0.172	1

Next, the evaluation on the criteria in selecting the coffee shops was done. The involved criteria were atmosphere (A_1), flavor (A_2), price (A_3), menu variety (A_4), service (A_5) and location (A_6). Table 3 represents the SFN for the criteria from five decision-makers.

Table 3 Criteria evaluation using SFN

	A_1	A_2	A_3	A_4	A_5	A_6
DM1	(0.9,0.1,0.1)	(0.9,0.1,0.1)	(0.9,0.1,0.1)	(0.9,0.1,0.1)	(0.9,0.1,0.1)	(0.9,0.1,0.1)
DM2	(0.7,0.3,0.3)	(0.9,0.1,0.1)	(0.9,0.1,0.1)	(0.9,0.1,0.1)	(0.9,0.1,0.1)	(0.9,0.1,0.1)
DM3	(0.7,0.3,0.3)	(0.7,0.3,0.3)	(0.8,0.2,0.2)	(0.9,0.1,0.1)	(0.9,0.1,0.1)	(0.9,0.1,0.1)
DM4	(0.7,0.3,0.3)	(0.9,0.1,0.1)	(0.6,0.4,0.4)	(0.8,0.2,0.2)	(0.9,0.1,0.1)	(0.8,0.2,0.2)
DM5	(0.9,0.1,0.1)	(0.9,0.1,0.1)	(0.8,0.2,0.2)	(0.8,0.2,0.2)	(0.9,0.1,0.1)	(0.7,0.3,0.3)

Next, the aggregation of the SFN, calculated using Eq. (3) and is represented in Table 4 together with their score functions and normalized weights.

Table 4 Aggregation of SFN, score function and normalized weight

Criteria	Aggregation	Score Function	Normalized Weight
A_1	(0.764,0.239,0.252)	4.684	0.130
A_2	(0.876,0.126,0.136)	6.546	0.182
A_3	(0.804,0.200,0.222)	5.292	0.147
A_4	(0.861,0.140,0.145)	6.298	0.175
A_5	(0.900,0.100,0.100)	7.020	0.195
A_6	(0.852,0.150,0.160)	6.120	0.170

Upon obtaining the normalized weights of criteria, three coffee shops in Jengka (Alt_1, Alt_2 and Alt_3) were ranked using simple additive weighting (SAW) and weighted product method (WPM). The aggregated decision matrix is represented by Table 5.

Table 5 Criteria evaluation using SFN

	Alt_1	**Alt_2**	**Alt_3**
A_1	(0.69,0.31,0.33)	(0.77,0.24,0.26)	(0.76,0.25,0.27)
A_2	(0.56,0.44,0.45)	(0.57,0.47,0.30)	(0.78,0.22,0.24)
A_3	(0.54,0.47,0.41)	(0.46,0.55,0.40)	(0.69,0.31,0.32)
A_4	(0.66,0.34,0.36)	(0.66,0.35,0.32)	(0.69,0.31,0.32)
A_5	(0.72,0.29,0.30)	(0.62,0.41,0.28)	(0.85,0.15,0.16)
A_6	(0.71,0.30,0.30)	(0.59,0.41,0.42)	(0.81,0.20,0.24)

Next, the spherical fuzzy weights were calculated using the SAW and WPM, and their score functions are evaluated as shown in Table 6.

Table 6 Spherical fuzzy weights and their score functions

	Simple Additive Weighted (SAW)		**Weighted Product Method (WPM)**	
	Spherical fuzzy weight	**Score function**	**Spherical fuzzy weight**	**Score function**
Alt_1	(0.66,0.35,0.36)	3.166	(0.64,0.37,0.37)	3.004
Alt_2	(0.62,0.40,0.33)	2.840	(0.60,0.42,0.34)	2.621
Alt_3	(0.78,0.23,0.25)	4.858	(0.77,0.24,0.26)	4.682

Results and Discussion

The alternatives were ranked based on the score functions, in which the alternative with the highest value is the most preferred coffee shop among the decision-makers. In reference to Table 6, the ranking of coffee shops using the spherical SAW and WPM is as follows: $Alt_3 \succ Alt_1 \succ Alt_2$.

To validate the consistency and stability of the developed decision-making model, the sensitivity analysis was performed by changing the weights of selected criteria. In this situation, the weights of criteria with the highest weightage (A_5) and lowest weightage (A_1) increase and decreases, respectively, by 50%. For this purpose, Eq. (8) is adopted from Memariani et al. (2009).

$$\varpi'_j = \begin{cases} \varpi_j + \delta_k & \text{if} \quad j = k \\ \left(\dfrac{1-\varpi'_k}{1-\varpi_k}\right)\varpi_j & \text{if} \quad j \neq k, j = 1,2,\ldots,m \end{cases} \tag{8}$$

where ϖ_j is the initial weight of the j-th criterion, ϖ'_j is the new weight of the j-th criterion, and δ_k is the increment or decrement of weight.

Tables 7 and 8 display the score functions of alternatives obtained after increasing and decreasing the criteria weight up to 50% for the SAW and WPM models, respectively.

Table 7 Results of sensitivity analysis for SAW model

	Increasing weight of A_1					**Decreasing weight of A_5**				
	10%	20%	30%	40%	50%	10%	20%	30%	40%	50%
Alt_1	3.203	3.239	3.274	3.310	3.345	3.183	3.200	3.217	3.233	3.250
Alt_2	2.896	2.952	3.006	3.061	3.115	2.864	2.888	2.912	2.936	2.959
Alt_3	4.887	4.915	4.943	4.971	4.999	4.882	4.905	4.928	4.951	4.974

Table 8 Results of sensitivity analysis for WPM model

	Increasing weight of A_1					**Decreasing weight of A_5**				
	10%	20%	30%	40%	50%	10%	20%	30%	40%	50%
Alt_1	2.981	2.958	2.935	2.913	2.891	2.976	2.948	2.921	2.894	2.867
Alt_2	2.609	2.597	2.585	2.573	2.561	2.600	2.580	2.559	2.538	2.518
Alt_3	4.649	4.616	4.584	4.552	4.520	4.660	4.639	4.617	4.596	4.575

Even the criteria weights are increasing or decreasing up to 50%, the developed SAW and WPM models maintain their consistency in producing the ranking of alternatives since the initial ranking is not changed. This result highlights the stability of the models in evaluating criteria weights and ranking the alternatives. However, comparing these results to the initial score functions obtained in Table 6 highlights that the WPM model performs better than the SAW model. Comparing the average deviation of each value from Table 7 and Table 8 using the mean square error, the SAW model produces an average error of 0.0120, meanwhile the WPM model produces an average error of 0.0061, which is much smaller than the SAW.

Conclusion

The simple additive weighting (SAW) and weighted product method (WPM) exhibited simplicity in fuzzy decision-making. This feature allowed ranking of alternatives in efficient and optimal time. The integration of the two approaches with spherical fuzzy sets (SFS) enhanced the handling of uncertainty and vagueness among decision-makers. This is due to the fact that SFS has larger domains of hesitancy and is independent of the membership and non-membership grades. Based on these aforementioned facts, this paper successfully implemented the spherical SAW and WPM approaches in determining the best coffee shops in Jengka, Pahang. Among the six criteria evaluated, service was the most preferred criterion in selecting a certain coffee shop, followed by flavor, menu variety, location, price, and atmosphere. The sensitivity analysis was performed by increasing and decreasing the criteria weights up to 50% and the results showed that the implemented approaches produced stable ranking of coffee shops. In fact, the WPM model performed better than the SAW model since it produced smaller errors in terms of score functions when compared to the initial values. In the future, other fuzzy multi-criteria decision-making approaches such as WASPAS, EDAS and ARAS can be adopted to assess customers' preferences in selecting the best coffee shops.

Ethics Statement

The research does not require research ethics approval.

Authors Contribution

Original draft preparation, Nik Badrul Alam, N.M.F.H.; Methodology, Nik Badrul Alam, N.M.F.H.; Discussion, Ramli, N.; Writing – Review and editing, Ramli, N.

Acknowledgement

The authors would like to thank Universiti Teknologi MARA (UiTM) Pahang for supporting this research.

Conflict of interests

All authors declare that there is no conflict of interests.

References

Akram, M., Zahid, K., & Kahraman, C. (2023). A PROMETHEE based outranking approach for the construction of Fangcang shelter hospital using spherical fuzzy sets. *Artificial Intelligence in Medicine*, *135*(November 2022), 102456. https://doi.org/10.1016/j.artmed.2022.102456

Aliev, R. A., Guirimov, B. G., Huseynov, O. H., & Aliyev, R. R. (2021). A consistency-driven approach to construction of Z-number-valued pairwise comparison matrices. *Iranian Journal of Fuzzy Systems*, *18*(4), 37–49.

Aydoğdu, A., & Gül, S. (2022). New entropy propositions for interval-valued spherical fuzzy sets and their usage in an extension of ARAS (ARAS-IVSFS). *Expert Systems*, *39*(4). https://doi.org/10.1111/exsy.12898

Aydoğdu, E., Güner, E., Aldemir, B., & Aygün, H. (2023). Complex spherical fuzzy TOPSIS based on entropy. *Expert Systems with Applications*, *215*(November 2021). https://doi.org/10.1016/j.eswa.2022.119331

Fauzi, A., Indriyani, N., & Bayu Hasta Yanto, A. (2021). Selection of Coffee Shop Business Locations Using the Analytical Hierarchy Process Method. *Jurnal Teknologi Dan Open Source*, *4*(2), 133–140. https://doi.org/10.36378/jtos.v4i2.1771

Garg, H., & Sharaf, I. M. (2022). A new spherical aggregation function with the concept of spherical fuzzy difference for spherical fuzzy EDAS and its application to industrial robot selection. *Computational and Applied Mathematics*, *41*(5), 1–26. https://doi.org/10.1007/s40314-022-01903-5

Hutagalung, J., Sari, V. W., & Aini, Z. (2023). Selection of the Best Coffee Shop Using the VIKOR Method. *JURTEKSI (Jurnal Teknologi Dan Sistem Informasi)*, *9*(4), 605–612. https://doi.org/10.33330/jurteksi.v9i4.2500

Kutlu Gündoğdu, F., & Kahraman, C. (2019). Spherical fuzzy sets and spherical fuzzy TOPSIS method. *Journal of Intelligent & Fuzzy Systems*, *36*(1), 337–352. https://doi.org/10.3233/JIFS-181401

Kutlu Gündoğdu, F., & Yörükoğlu, M. (2021). Simple Additive Weighting and Weighted Product Methods Using Spherical Fuzzy Sets. In *Advances in Intelligent Systems and Computing: Vol. 1197 AISC* (pp. 241–258). https://doi.org/10.1007/978-3-030-45461-6_10

Memariani, A., Amini, A., & Alinezhad, A. (2009). Sensitivity Analysis of Simple Additive Weighting Method (SAW): The Results of Change in the Weight of One Attribute on the Final Ranking of Alternatives. *Journal of Industrial Engineering*, *4*, 13–18.

Purba, R. R., Mesran, M., Zaen, M. T. A., Setiawansyah, S., Siregar, D., & Ambarsari, E. W. (2023). Decision Support System in the Best Selection Coffee Shop with TOPSIS Method. *The IJICS (International Journal of Informatics and Computer Science)*, *7*(1), 28. https://doi.org/10.30865/ijics.v7i1.6157

Sharaf, I. M. (2021). Global Supplier Selection with Spherical Fuzzy Analytic Hierarchy

Process. In C. Kahraman & F. Kutlu Gündoğdu (Eds.), *Decision Making with Spherical Fuzzy Sets* (pp. 323–348). Springer. https://doi.org/10.1007/978-3-030-45461-6_14

Wang, C. N., Nguyen, N. A. T., & Dang, T. T. (2022). Offshore wind power station (OWPS) site selection using a two-stage MCDM-based spherical fuzzy set approach. *Scientific Reports*, *12*(1), 1–21. https://doi.org/10.1038/s41598-022-08257-2

Windarto, A. P., Mesran, Saidah, F., & Ambarsari, E. W. (2024). Implementation of the Preference Selection Index (PSI) Method in Courier Partner Recruitment. *Bulletin of Artificial Intelligence*, *3*(1), 35–41. http://dx.doi.org/10.33330/jurteksi.v10i3.3212Availableonlineathttp://jurnal.stmikroyal.ac.id/index.php/jurteksi

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X