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PATH FINDING OF INDOOR MOBILE ROBOT USING HARMONIC POTENTIALS VIA EXPLICIT DECOUPLED GROUP MODIFIED ACCELERATED OVER RELAXATION METHOD

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ABSTRACT

The harmonic potential fields, a solution to equation of Laplace are widely used in robot path finding as a suggestion for robot course-plotting in an identified environment. The computation of these harmonic functions often involves simulations on high-performance computer. In pursuit to solve the problem of robot navigation, this article suggests a technique called Half-Sweep Block Modified Accelerated Over-Relaxation or better known as Explicit Decoupled Group Modified Accelerated Over-Relaxation (EDGMAOR). To verify the effectiveness of EDGMAOR, simulations of robot navigation were applied in a static known enclosed environment. Experiments are provided to assess the performance of the suggested technique. In particular, different starting and goal positions are used to assess the paths generated from the simulations. The outcomes show the advantages of the proposed algorithm. In the end, the research indicates that the proposed method in computing harmonic functions is appealing and attainable for solving path planning problem.

Keywords: Laplace's Equation; Collision Free; Optimal Path; Iterative Method

INTRODUCTION

Among of arduous task in robotics application is to develop intelligent autonomous motion planning. Currently, an intelligent autonomous robot has an increasing demand in various applications such as space, transportation, industry, defense and security settings. A success autonomous mobile robot should be efficient and reliable in planning a route from a starting position to a destination position without robot hitting the obstacles. This article introduces the issue of a mobile robot path finding expressed as a heat transfer analogy. The heat transmission is illustrated by the equation of Laplace's. One of the important features of heat transfer is the ability to overcome the issue of local minima, make it very promising in robot navigation control.

In the past, most path generation approaches using harmonic potentials relied on typical 5-point (5P) Laplacian. The attainment of a skewed 5P Laplacian is investigated along with several experiments in this study. The resulting skewed operator developed a scheme of algebraic skewed approximation equations. The article presents a new technique for computing algebraic skewed linear systems. The technique is named Explicit Decoupled Group Modified Accelerated Over-Relaxation (EDGMAOR). The performance of the proposed technique in calculating harmonic functions for solving the problem of path planning is compared against existing algorithm. Numerical simulations demonstrate that the EDGMAOR outperform the existing methods with the least computational time and iteration counts.

Harmonic Function in Path Navigation

On domain $\Omega \subset R^n$, a harmonic function that fulfill the Laplace's equation as follow

$$\nabla^2 \phi = \sum_{i=1}^{n} \frac{\partial^2 \phi}{\partial x_i^2} = 0 \qquad (1)$$

with x_i indicates the $i-$th coordinates of Cartesian and the dimension be represented as n. In subject to generate path planning, the domain Ω consists of several components such as the walls, borders, obstacles/hindrances/barriers, multiple initial points and target point. The formation of a false local minimum in the domain Ω is prevented if Eq. 1 is exploited to limit the functions generated in the environment. This is due to the advantages of the harmonic functions which fulfill the minimum-maximum condition. Hence harmonic functions are very practical in robot path finding as they provide outright path algorithm which will ensure smooth and efficient autonomous robot navigation [1]. It is known in the literature that Laplace's equation was computed numerically by means of ordinary approaches [2-4] for instance Jacobi**,** Gauss Seidel (GS), and Successive Over-Relaxation (SOR). In pursuit to speed up the computation, this article aims to define Eq. 1 via accelerated iterative method named EDGMAOR.

Essentially the potential field is calculated globally in path finding problem. Solving Laplace expression as in Eq. 1 resulted to the harmonic function. It is employed to find a route that moves the point robot gradually from initial to the target position point while never touching any obstacle. The hindrances are always measured as sources of new while the target is expressed as the sink, which has set of lowermost potential value. This adds up to the application of Dirichlet boundary conditions. Later, by carry out the Gradient Descent Search (GDS) on the potential field, a series of potential points through lower values will guide to the lowermost value which is the goal point, to be discovered.

To facilitate in describing the outcome of Laplace expression Eq. 1, this study applied the above mentioned framework for path planning problem. The idea behind is by applying the concept of how temperature and heat stream works in generating potential value and path line for robot navigation. The experiment is performed on 2D domain with different form of obstacles. The proposed iterative scheme, EDGMAOR is employed to compute Laplace's Equation as well as to acquire the values of temperature at every node. To evaluate its performance, comparisons with following iterative methods were taken into account: the standard Explicit Group Successive Over-Relaxation (EGSOR), Explicit Group Accelerated Over-Relaxation (EGAOR), Explicit Group Modified Successive Over-Relaxation (EGMSOR), Explicit Group Modified Accelerated Over-Relaxation (EGMAOR), Explicit Decoupled Group Successive Over-Relaxation (EDGSOR), Explicit Decoupled Group Accelerated Over-Relaxation (EDGAOR) and Explicit Decoupled Group Modified Successive Over-Relaxation (EDGMSOR).

Explicit Decoupled Group Modified Accelerated Over-Relaxation Scheme

In 1991, Abdullah [5] was first to initiate the Explicit Decoupled Group (EDG) iterative approach, also known with the Half-Sweep of block iterative technique in unravelling equation of 2-dimensional Poisson. Since at that moment, EDG method has been implemented to find the solution of partial differential equations [6-12] while the modified version is explored in resolving diffusion equation [13]. Preliminary attempts have been identified [14] in merging SOR accompanied by other practice. In the literature, ordinary GS [15] and SOR [16, 17] were executed for solving Eq. (1). This study suggested a lesser computational timings solver by means of an accelerated iteration technique, called Modified Accelerated Over-Relaxation (MAOR) iterative technique, seeking the solutions for Eq. 1. Let the equation of 2-dimensional Laplace's as per Eq. 1 signified the following

$$\nabla^2 U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0\,. \tag{2}$$

The basic 5P finite difference method for Eq. 2 are essentially referred to the second-order central difference approximation, commonly expressed as

$$U_{i-1,j} + U_{i+1,j} + U_{i,j-1} + U_{i,j+1} - 4U_{i,j} = 0 \tag{3}$$

Intended for the Laplace's Eq. 2, this approach principally contains of repetitively swapping every value of the node with an average of its four neighbor nodes. These nodal values are set fixed that shall be regarded as the internal and external borders, barriers and the target point.

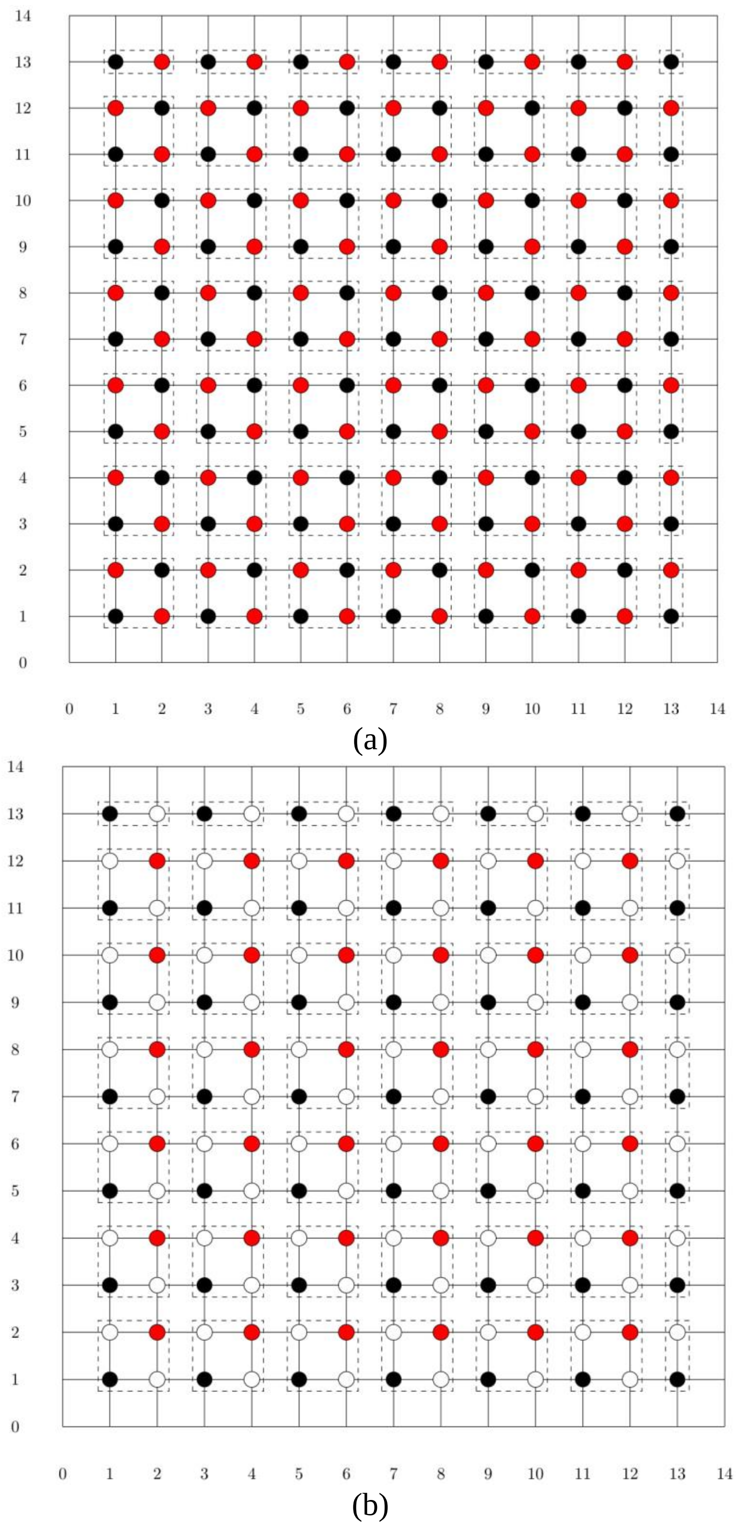
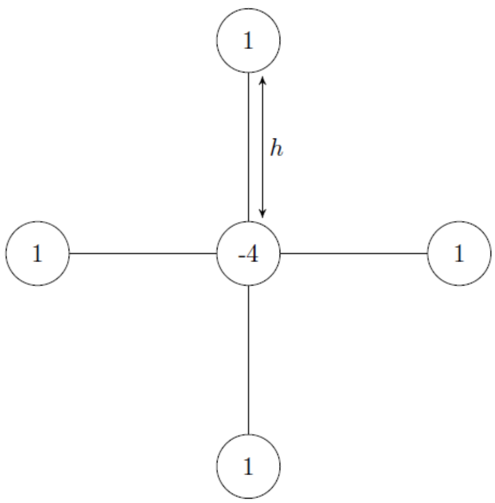
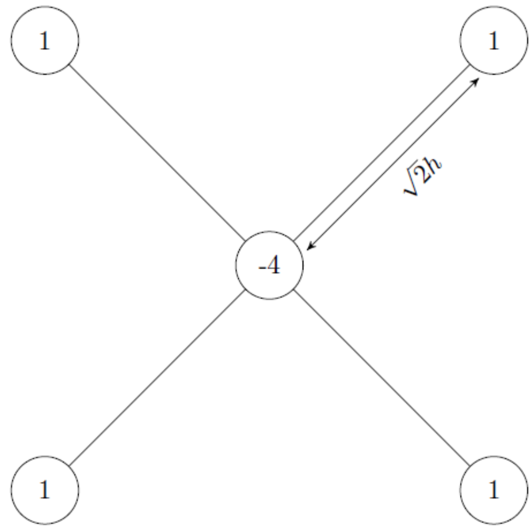


Figure 1: The computational particles of EDG for (a) full-sweeps and (b) half-sweep iterations

Instead of applying the EDG approach to the entire points, it is applied to a reduced point. Hence the computation involved is greatly reduced. As shown in Figure 1(b), the execution of EDG iteration are carried out on groups of pair of red-black nodes only per block contrasted with the full-sweep technique which iterates all red-black points as Figure 1(a). The half-sweep approach was first utilized by [18]. The prominent properties of EDG iteration of considering reduced node points resulted into much simpler computation. The stencil or pattern of full-sweep (standard) and half-sweep (rotated) iteration practices are demonstrated in following Figure 2.



(a)



(b)

Figure 2: The computational patterns for (a) standard and (b) rotated cases

Fundamentally, the approximation equation from EDG is in fact constructed on an intersect alignment operator that seemingly reached in clockwise direction by rotating 45° of the *i-j* plane axis. This produce the rotated pattern or also called as skewed 5P approximation formula define as

$$U_{i-1,j-1}+U_{i+1,j-1}+U_{i-1,j+1}+U_{i+1,j+1}-4U_{i,j}=0\,. \tag{4}$$

Once the weighted parameter from SOR method [19,20] is augmented towards Eq. 4, the execution of EDG iteration could be stated as

$$U_{i,j}^{(k+1)}=\frac{\omega}{4}\left(U_{i-1,j-1}^{(k+1)}+U_{i+1,j-1}^{(k+1)}+U_{i-1,j+1}^{(k)}+U_{i+1,j+1}^{(k)}\right)+\left(1-\omega\right)U_{i,j}^{(k)}\,. \tag{5}$$

To enhance the computational progress, a method of MAOR was employed into Eq. 4. Although the design for MAOR iterative scheme looks almost analogous to AOR technique, the MAOR schemes actually contains three distinct weighted parameters i.e. r,ω and ω'. The formula of MAOR method, which concerning red-black ordering strategy, shown as

$$U_{i,j}^{(k+1)}=\frac{\omega}{4}\left[U_{i-1,j+1}^{(k)}+U_{i+1,j+1}^{(k)}+U_{i-1,j-1}^{(k)}+U_{i+1,j-1}^{(k)}\right]+\left(1-\omega\right)U_{i,j}^{(k)}\,, \tag{6}$$

$$U_{i,j}^{(k+1)} = \frac{r}{4}\left[U_{i-1,j+1}^{(k+1)} - U_{i-1,j+!}^{(k)} + U_{i-1,j-1}^{(k+1)} - U_{i-1,j-1}^{(k)}\right] + \frac{r}{4}\left[U_{i+1,j+1}^{(k+1)} - U_{i+1,j+1}^{(k)} + U_{i+1,j-1}^{(k+1)} - U_{i+1,j-1}^{(k)}\right] + \frac{\omega'}{4}\left[U_{i-1,j+1}^{(k)} + U_{i+1,j+1}^{(k)} + U_{i-1,j-1}^{(k)} + U_{i+1,j-1}^{(k)}\right] + (1-\omega')U_{i,j}^{(k)}, \quad (7)$$

where Eq. 6 is for red nodes while Eq. 7 is for black nodes, in which r, ω and ω' are the optimal parameters as characterise in $[1,2)$. Until now, obtaining the minimum value of the optimal parameters does not have a general formula. Normally, the number of r and ω' is carefully chosen to be near to ω of the corresponding SOR [21].

EXPERIMENTS AND RESULTS

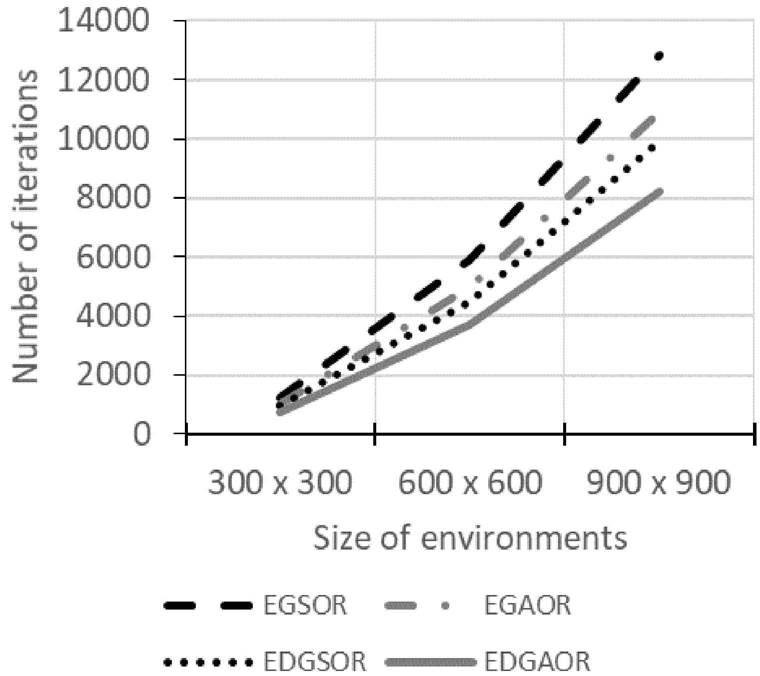
The area of environment used for simulations were 300x300, 600x600 and 900x900. Different amount of hindrances with mixed shapes have been set in the configuration space. The obstacles and all the walls were initially set at high temperature values. In the meantime, the target point has been set to be a very low temperature while the starting point does not have any initial value. Zero temperature value was set to all other points. The experiments were carried out on a personal computer executing at 2.50GHz speed with RAM 8GB. While waiting for the stopping criterion satisfied, the looping progression continued to calculate the temperature values at every points. Specifically, as soon as there has no alteration in the temperature value from one iteration process to the next or converged to a specified value 1.0^{-10}. Such setting is necessary to avert from flat areas which lead to failure in generating path to attain the target point. Tables 1 and 2 respectively reported the iteration counts and computing time (in seconds) taken to compute every temperature values in the configuration space for 8 different procedures involved for comparisons in the experiment. Figure 3(a) and 4(a) illustrated performance for standard method while Figure 3(b) and 4(b) illustrated performance for modified method. The proposed method, EDGMAOR demonstrated to have the least computing time with less number of iterations needed (refer Figure 3(b) and 4(b)) in comparison with other existing methods. Clearly, it proved to be the fastest among all.

Table 1: Iteration counts of different methods subject to 3 dimensions

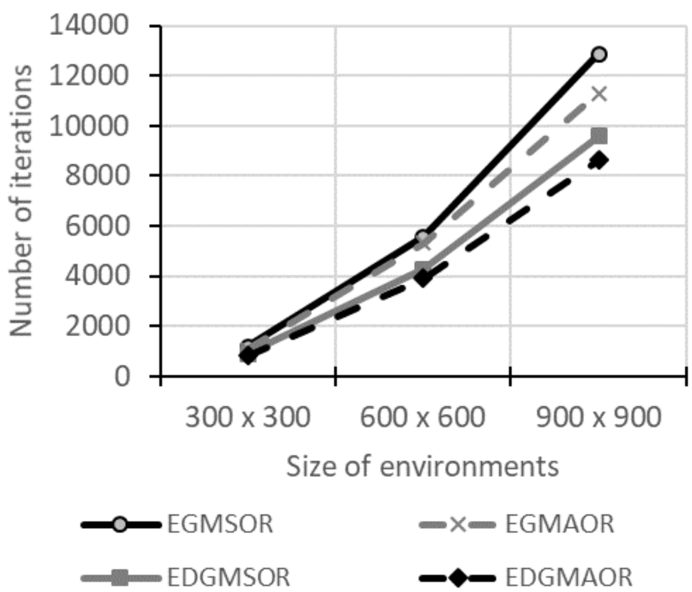
		Dimension		
	Methods	**300 x 300**	**600 x 600**	**900 x 900**
Case 1	EGSOR	1258	5899	12844
	EGAOR	1042	4994	10928
	EGMSOR	1182	5552	12871
	EGMAOR	1079	5305	11289
	EDGSOR	953	4495	9916
	EDGAOR	766	3730	8200
	EDGMSOR	898	4283	9585
	EDGMAOR	816	3921	8619
Case 2	EGSOR	1729	6782	14874
	EGAOR	1610	6368	13953
	EGMSOR	1657	6559	15276
	EGMAOR	1557	6154	13543
	EDGSOR	1324	5599	13252
	EDGAOR	1188	4767	10490
	EDGMSOR	1252	5456	14417
	EDGMAOR	1175	4733	10416
Case 3	EGSOR	2666	11076	24519
	EGAOR	2480	10389	22995
	EGMSOR	2552	10709	23608
	EGMAOR	2370	10002	21996
	EDGSOR	2035	10243	24864
	EDGAOR	1835	7741	17131
	EDGMSOR	1989	10238	23681
	EDGMAOR	1820	7663	16981
Case 4	EGSOR	1629	6487	14194
	EGAOR	1392	5648	12367
	EGMSOR	1568	6255	13701
	EGMAOR	1317	5433	12863
	EDGSOR	1238	5958	13501
	EDGAOR	1116	4223	9369
	EDGMSOR	1265	5702	13079
	EDGMAOR	1023	4214	9637

Table 2: Computing time (in seconds) of different methods subject to 3 dimensions

		Dimension		
	Methods	**300 x 300**	**600 x 600**	**900 x 900**
Case 1	EGSOR	6.88	163.72	871.66
	EGAOR	6.05	137.87	751.78
	EGMSOR	6.74	165.65	983.56
	EGMAOR	6.66	169.96	880.50
	EDGSOR	4.34	110.88	603.41
	EDGAOR	3.67	95.08	498.19
	EDGMSOR	4.73	117.61	631.16
	EDGMAOR	4.36	119.39	621.90
Case 2	EGSOR	7.67	199.59	1009.48
	EGAOR	8.25	185.36	926.49
	EGMSOR	7.99	205.17	1150.19
	EGMAOR	8.25	205.08	1022.43
	EDGSOR	4.70	142.64	787.53
	EDGAOR	4.17	122.85	624.48
	EDGMSOR	5.06	154.59	972.91
	EDGMAOR	5.25	146.83	734.06
Case 3	EGSOR	13.24	315.87	1602.81
	EGAOR	13.83	301.27	1633.35
	EGMSOR	12.64	334.98	1828.16
	EGMAOR	12.28	327.72	1801.35
	EDGSOR	8.00	261.74	1529.74
	EDGAOR	7.67	203.61	1072.61
	EDGMSOR	8.74	285.14	1686.18
	EDGMAOR	8.23	237.03	1291.76
Case 4	EGSOR	7.80	187.33	990.20
	EGAOR	7.56	167.65	891.51
	EGMSOR	6.94	196.36	1059.32
	EGMAOR	6.41	170.40	1041.04
	EDGSOR	4.33	152.73	828.24
	EDGAOR	4.53	109.96	615.38
	EDGMSOR	4.83	162.47	929.50
	EDGMAOR	4.70	133.39	745.72

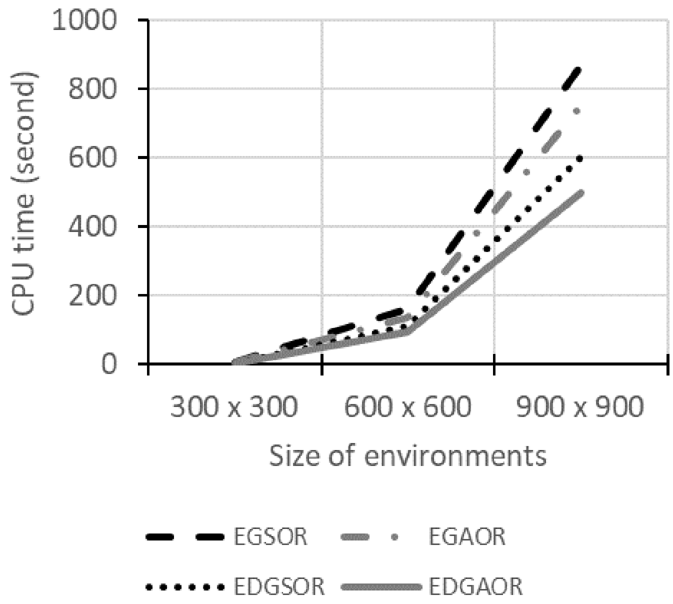


(a)

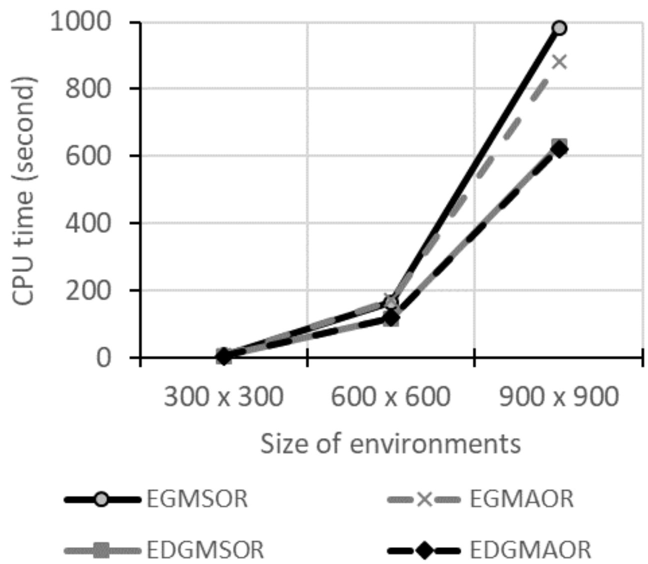


(b)

Figure 3: Iteration counts in varying environments



(a)



(b)

Figure 4: Computing time (in seconds) in varying environments

When the configuration space potential values are acquired, the trail generation takes place by employing steepest descent search beginning at any initial point to the specific target point. The algorithm tracks the negative gradient and repeatedly selects the lowest temperature around the neighborhood points up until the created path arrives at its target point. Figure 5 illustrates the successful path generation from numerical computation, in a known stationary environment. All of the start points manage to end at the specific target point while escaping numerous barriers in the given configuration space.

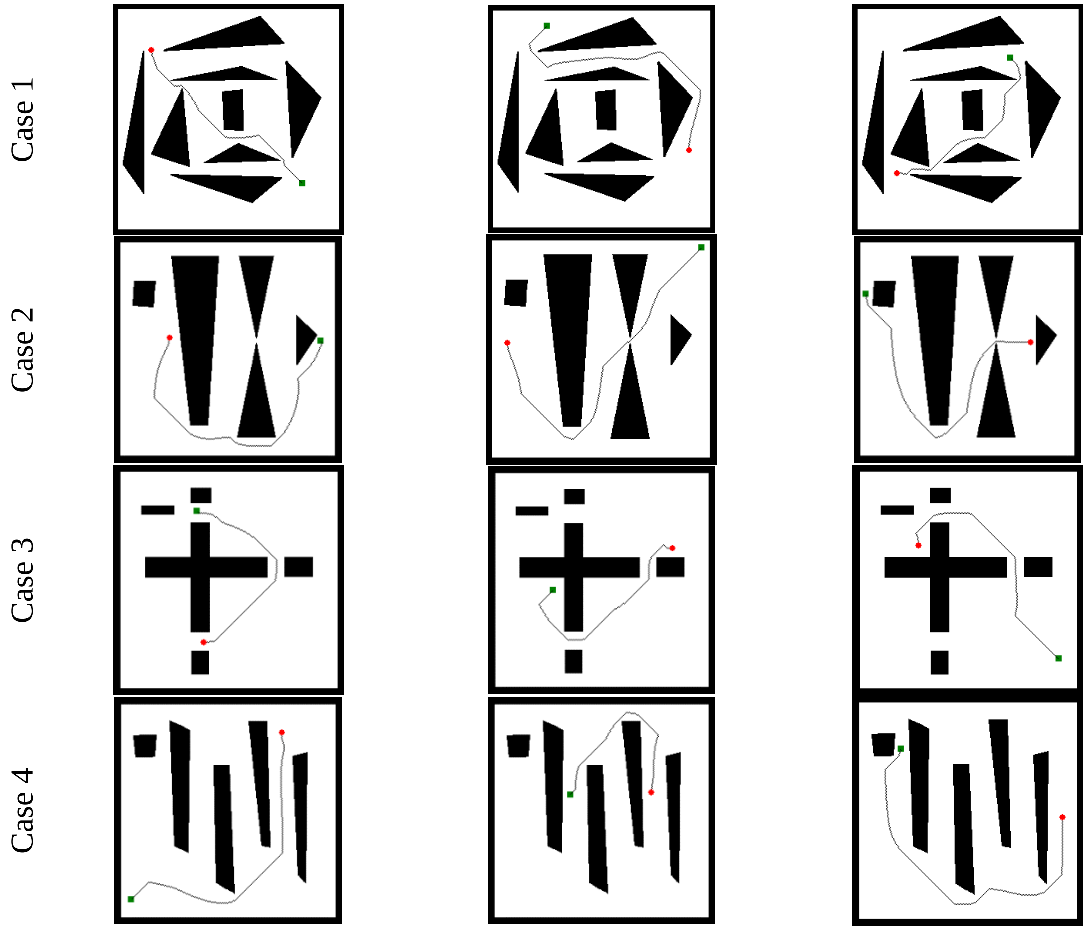


Figure 5: The generated paths from several different start (green dot/square dot) and goal positions (red dot/circle dot) for various environment

CONCLUSION

To put it briefly, the investigations in this study highlights that harmonic functions provide a promising and feasible way of creating pathways in a point robot environment attribute to advanced finding numerical techniques, together with the new advanced computing technologies. The simulation outcomes confirmed that the EDGMAOR iterative scheme is the fastest compared to the standard method (SOR and AOR). While the obstacles number has risen, the efficiency of the proposed method is not affected. In reality the calculation becomes quicker as the obstacles zones are omitted throughout the calculation process. Furthermore, the originality of this work is the use of the AOR scheme families in robot path finding. In addition to the idea of the full- and half-sweep techniques, additional–analysis may also be considered on quarter-sweep [22-24] iterations with the intention of accelerating the rate of convergence for the suggested iterative method.

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REFERENCES

[1] Connolly, C.I., and Gruppen, R. *J. Robot. Syst.* **10** (7) 931-946 (1993)

[2] Evans, D.J. *Int. J. Comp. Maths.* **17** 81-108 (1985)

[3] Evans, D.J., and Yousif, W.S. *Int. J. Comp. Math.* **18** 323-340 (1986)

[4] Ibrahim, A. (1993); The Study of The Iterative Solution of Boundary Value Problem by the Finite Difference Method, PhD thesis, Universiti Kebangsaan Malaysia, Malaysia.

[5] Abdullah, A.R. *Int. J. Comp. Math.* **38** 61-70 (1991)

[6] Ibrahim, A., and Abdullah, A.R. *Int. J. Comp. Math.* **58** 253-256 (1995)

[7] Dahalan, A.A., Muthuvalu, M.S., and Sulaiman, J. *AIP Conf. Proc.* 1557 (1) 103-107 (2013)

[8] Yousif, W.S., and Evans, D.J. *Parallel Algorithms Appl.* **7** 53-71 (2001)

[9] Dahalan, A.A., Muthuvalu, M.S., and Sulaiman, J. *Appl. Math. Sci.* **8** (18) 885-899 (2014)

[10] Abdullah, A.R., and Ali, N.H.M. *Parallel Algorithms Appl.* **10** 93-103 (1996)

[11] Dahalan, A.A., Muthuvalu, M.S., and Sulaiman, J. *U.P.B. Sci. Bull., Series A.* **77** (2) 251-260 (2015)

[12] Dahalan, A.A., Muthuvalu, M.S., and Sulaiman, J. *AIP Conf. Proc.* **1682** (1) 020003 (2015)

[13] Sulaiman, J., Hassan, M.K., and Othman, M. *The half-sweep iterative Alternating Decomposition Explicit (HSIADE) method for diffusion equation, Lecture Notes in Computer Sciences 3314: Comp. And Info. Sci.,* 57-63 (2004)

[14] Sulaiman, J., Othman, M., and Hassan, M.K. (2009); Nine Point-EDGSOR iterative method for the finite element solution of 2D Poisson equations,bO. Gervasi et al. (Eds): ICCSA 2009, Part I, LNCS 5592, pp. 764-774 (Springer-Verlag Berlin Heidelberg).

[15] Connolly, C.I., Burns, J.B., and Weiss, R. (1990); Path planning using Laplace's equation, in Proceedings of IEEE International Conference on Robotics and Automation, pp. 2102-2106.

[16] Saudi, A., and Sulaiman, J. (2013); Robot path planning using Laplacian Behaviour-Based Control (LBBC) via Half-sweep SOR, in Proceedings of the International Conference on Technological Advances in Electrical, Electronics and Computer Engineering, Konya, Turkey. May 9-11, pp. 424 – 429.

[17] Sasaki, S. *Proc. IEEE Int. Conf. Control Appl.* 1323-1327 (1998)

[18] Saudi, A., and Sulaiman, J. (2009); Half-sweep Gauss-Seidel (HSGS) iterative method for robot path planning, in The 3rd International Conference on Informatics and Technology (Informatics09) in Kuala Lumpur.

[19] Young, D.M. *Trans. Amer. Math. Soc.* **76** 92-111 (1954)

[20] Young, D.M. (1971); Iterative Solution of Large Linear Systems, London: Academic Press.
[21] Hadjidimos, A. *Math. Comput.* **32** (141) 149-157 (1978)
[22] Othman, M., and Abdullah, A.R. *Int. J. Comput. Math.* **76** 203-217 (2000)
[23] Dahalan, A.A., and Sulaiman, J. Int. J. Math. Anal. **9** (35) 1733-1746 (2015)
[24] Sulaiman, J., Hasan, M. K., Othman, M., and Karim, S.A.A. *Procedia Comp. Sci.* **1** 377–385 (2010)