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STUDY OF SIMPLE PENDULUM USING TRACKER VIDEO ANALYSIS AND HIGH SPEED CAMERA: AN INTERACTIVE APPROACH TO ANALYZE OSCILLATORY MOTION

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ABSTRACT

In this paper, we report on the use of Tracker video analysis and high speed camera as an interactive approach to study oscillatory motion of a simple pendulum. Tracker software is basically a computer based learning tool and is preferred because it is free, user friendly and support effective learning and teaching. Combining with the high speed camera that records the motion of pendulum at a frame rate up to 1000 frames per second (fps), analysis of the motion is performed at different angles and video qualities. The periods obtained from the experiment are then compared with the exact period expression and Lima and Arun approximation in order to determine how well this approach suited for the large angle approximation. Results have shown that when the video qualities improved, errors are minimal but errors increased when the angle increased. This research finding shows that this approach is feasible in studying the motion of simple pendulum and at the same time, interactive and inexpensive.

Keywords: High-Speed Camera; Simple Pendulum; Tracker Video Analysis;

INTRODUCTION

In the mechanical world, the motion and oscillation of objects not only can be visualised, but also studied in details. While oscillatory motion like the simple pendulum experiment is a common lesson in pre-university and undergraduate level, one of the most daunting things to learn and understand in oscillatory motion is that students should be able to imagine and visualize the association between the movement of an object oscillated in oscillatory motion and the complex mathematical calculation of the movement. Therefore, to make sure that students are engaged in learning physics, teaching and learning should be interesting and up-to-date. One way to achieve that is by the integration of information technologies into the teaching and learning process.

This method is more cost effective as compared to sensor based data acquisition [1].

Video analysis in physics education has getting much attention nowadays [2] because visual display can make traditional lesson more intriguing [3] and more effective in teaching and learning process [2]. Tracker video analysis stands out as one of the new innovative approaches for teaching and learning physics where it makes natural sciences more interesting than ever before [4]. With Tracker, modeling and analysis of the motion of objects in videos are possible and easy to use simply by overlaying simple dynamical models onto the videos directly. This model will then automatically synchronize and scaled to the video for direct comparison with the real world.

As for the use of high speed video cameras, they have been found useful to discover and study multifarious mechanical motions [5]. High speed cameras can well capture various images and recording videos at the rate of more than 100 frames per second (fps) [6], compared to the normal camera which is played back at 25 or 30 fps. The use of high speed video technology allows us to record video images of very fast motion normally not visible to human eye and the motion can be visualized by having them replayed in slow motion [7]. Some of the example of high-speed video camera-based experiments is in the studies of bungee jumping [8], damped harmonic motion [9] and the falling of shuttlecock [5].

Many different measurements has been performed to find accurate period measurement for large angle oscillation for simple pendulum [10], but somehow they felt short from the exact period solution and Lima & Arun approximation (L&A). In this project, we make use of the high speed video camera and Tracker software as the main tools to collect reliable data for large angle oscillation. This work prioritises the study of the period of oscillation on various angles and video qualities. The periods are then calculated based on the damped angular frequency retrieved from the fitted curved in Trackers. A small time interval is considered in order to reduce damping caused by any frictional forces. These results are then compared with the work of L&A expression as well as the exact period expression for large angle approximation to check on the feasibility of this approach.

EXPERIMENTAL

Large angle oscillation

A simple pendulum consists of a small mass (m), also known as the pendulum bob suspended from an inextensible string of negligible mass to a pivot. When the bob is released with an initial angle (θ), the oscillation of a simple pendulum can be described using the following differential equation [11]:

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0 \qquad (1)$$

where g is the acceleration of gravity, L is the length of pendulum and θ is the amplitude of the angular displacement. The solution to equation (1) is expressed in terms of the elliptic integrals and there is no analytical solution to this differential equation. To solve it, one can either use numerical method or approximation. The simplest form of approximation is by considering a very small angle whereby $\sin\theta$ can be approximated by θ ($\sin\theta \approx \theta$). Hence, equation (1) is reduced to a linear equation:

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0 \qquad (2)$$

where the period is $T_0 = 2\pi\sqrt{L/g}$. For the exact period solution [12], it is given as

$$T_{ex} \approx T_0\left(1 + \frac{1}{16}\theta^2 + \frac{11}{3072}\theta^4 + \frac{173}{737280}\theta^6 + \frac{22931}{1321205760}\theta^8 + \cdots\right) \qquad (3)$$

where the elliptic integral of the first kind is expanded in power series. This solution usually is termed as the theoretical value for large angle oscillation. As for large angle approximation, work by Lima and Arun is considered due to its relatively small errors; less than 0.1% for angle smaller than 30° and less than 0.3% for angle smaller than 50° [12]. The approximated period is given as

$$T_{LA} = T_0\left(\frac{\ln\left(\cos\left(\frac{\theta}{2}\right)\right)}{1 - \cos\left(\frac{\theta}{2}\right)}\right) \qquad (4)$$

Damped Oscillation of Simple Pendulum

In reality, the pendulum cannot oscillate forever. Resistive forces such as air resistance and frictional force between string and pivot will diminish the amplitude of oscillation. We ignored the frictional force due to negligible mass of the string and only consider the air resistance, $f = -bv = -b\dot{x}$ where b is a constant. Couple with linear restoring force $-kx$, from Newton's second law, the equation of motion for the damped oscillation is $m\ddot{x} + b\dot{x} + kx = 0$ and can be rewritten as

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0 \qquad (5)$$

Here $\beta \equiv b/2m$ is the damping parameter and $\omega_0 = \sqrt{k/m}$ is the characteristic angular frequency in the absence of damping. For the underdamped motion $(\omega_0^2 > \beta^2)$, the general solution is given as

$$x(t) = Ae^{-\beta t}\cos(\omega_1 t - \delta) \qquad (6)$$

where $e^{-\beta t}$ is the damping factor, $Ae^{-\beta t}$ is the damped amplitude and $\omega_1^2 = \omega_0^2 - \beta^2$ is the damped angular frequency. Period oscillation of simple pendulum is given by:

$$T_{track} = \frac{2\pi}{\omega_1} \tag{7}$$

When the damping is small, $\omega_1 \approx \omega_0$, equation (6) will be used to fit our curve in Tracker.

METHODOLOGY

The experiment is carried out in a laboratory where fan and air condition are switched off to ensure that the air resistance is minimal. A simple and repeatable experiment is set up by attaching a spherical bob with a mass of 68.72 g to a light string of length 0.46 m from a rigid support at the wall as shown in Figure 1. To enhance the colour contrast and increase the video analysis accuracy, the bob is painted white and the black background is used to improve the view of the bob in the video. An external lighting is also provided to brighten up the experiment. A high-speed camera (Sony DSC-RX100M4) serves to record videos of the bob oscillation at four different frame rates which are 25 fps (normal rate), 250 fps, 500 fps and 1000 fps. For each frame rate, the pendulum bob is released at five different initial angles particularly 10°, 20°, 30°, 40° and 50°. The camera is placed between 0.5-1.0m away from the background before each trial. The camera is arranged perpendicular to the plane of oscillation simply to minimize systematic errors that might occur during experiment. While recording the video, we can adjust the focus of the camera to ensure that the observed pendulum oscillation is captured at the focal plane of camera. This will provide us with sharp images.

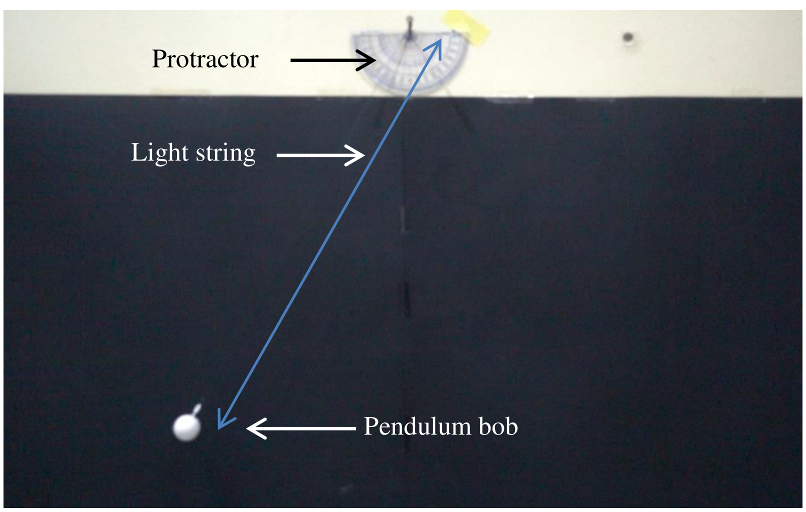

Figure 1: The experiment setup viewed with the high-speed camera

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VIDEO ANALYSIS

The videos from the experiment are then examined using Tracker to study the motion of simple pendulum. The video formats supported by Tracker software are Audio Video Interleave (AVI) and also MP4.

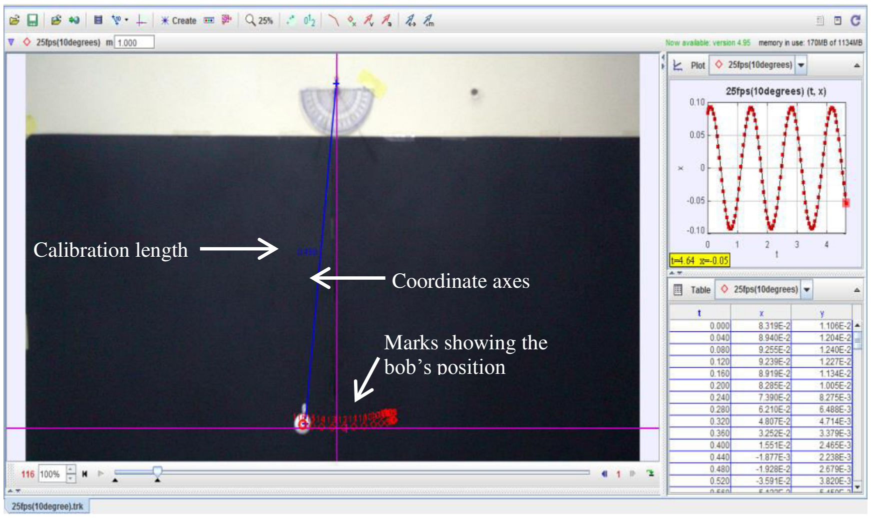

Figure 2: Video analysis of simple pendulum using Tracker

After importing the video to Tracker, we need to set the coordinate system and also to pinpoint the length of pendulum. Equilibrium position of the pendulum is set as the origin while the displacement of the string as shown in Figure 2 will provide us with the angle. We then set the bob as a point mass so that the oscillation motion in the video can be autotrack by Tracker. The autotracking function in Tracker allows accurate tracking without having to use a mouse. We used the fit builder in the data tool to analyse the graph which is shown in Figure 3. The motion of pendulum as shown in Figure 3 is an underdamped oscillation and the fitted equation is given by

$$x = Ae^{-Bt}\sin(Ct + D) + E \qquad (8)$$

where $Ae^{-\beta t}$ is the damped amplitude , B is the damping parameter, *C* is the damped angular frequency, *D* is the phase angle and *E* is the x-position shift parameter. It is found that parameter B from each frame rate is basically in order of -3 or in another words, the damping is negligible, $\omega_1 \approx \omega_0$. Hence, the period T_{track} calculated from our approach can be used to compare with the large angle approximation.

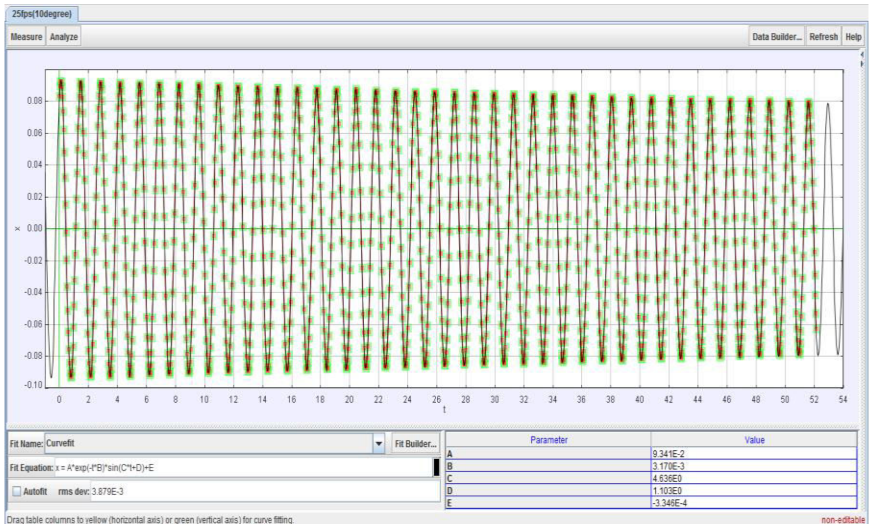

Figure 3: Analysis of the under-damped oscillation of pendulum using Data Tool (Curve Fitting)

RESULT AND DISCUSSION

The periods of the oscillation calculated using exact period solution (equation 3), L&A approximation (equation 4) and based on Tracker video analysis approach are shown in Table 1. We found that when the video qualities improved particularly from 25 fps to 1000 fps, our video analysis approach is entirely consistent with L&A approximation as shown in Figure 4. In our analysis, we only compare with L&A approximation because its formula is easier (suitable for students) and at the same time the periods calculated are basically similar with the exact period solution except at 50° with a relative error of 0.02%.

From Table 1, the result at normal rate shows the greatest error of 0.578% at 10° and gradually increased to 2.002% at 50°. These errors are mainly caused by motion blurring effect which link to the video quality. When a video is recorded at 25fps, each frame has an exposure time up to 40ms (1/25 seconds). All the motion of the bob in the scene over that entire 40ms is blended into the final frame. Hence, the oscillation in the video became blurring as the bob moves with high velocity at large initial angle. Due to Tracker feature that requires a precise figure template previously captured on the pendulum bob, so if the frame rate of video is too low, the tracker will no longer be able to obtain the best result because it failed to pin point accurately the point mass set at the bob earlier.

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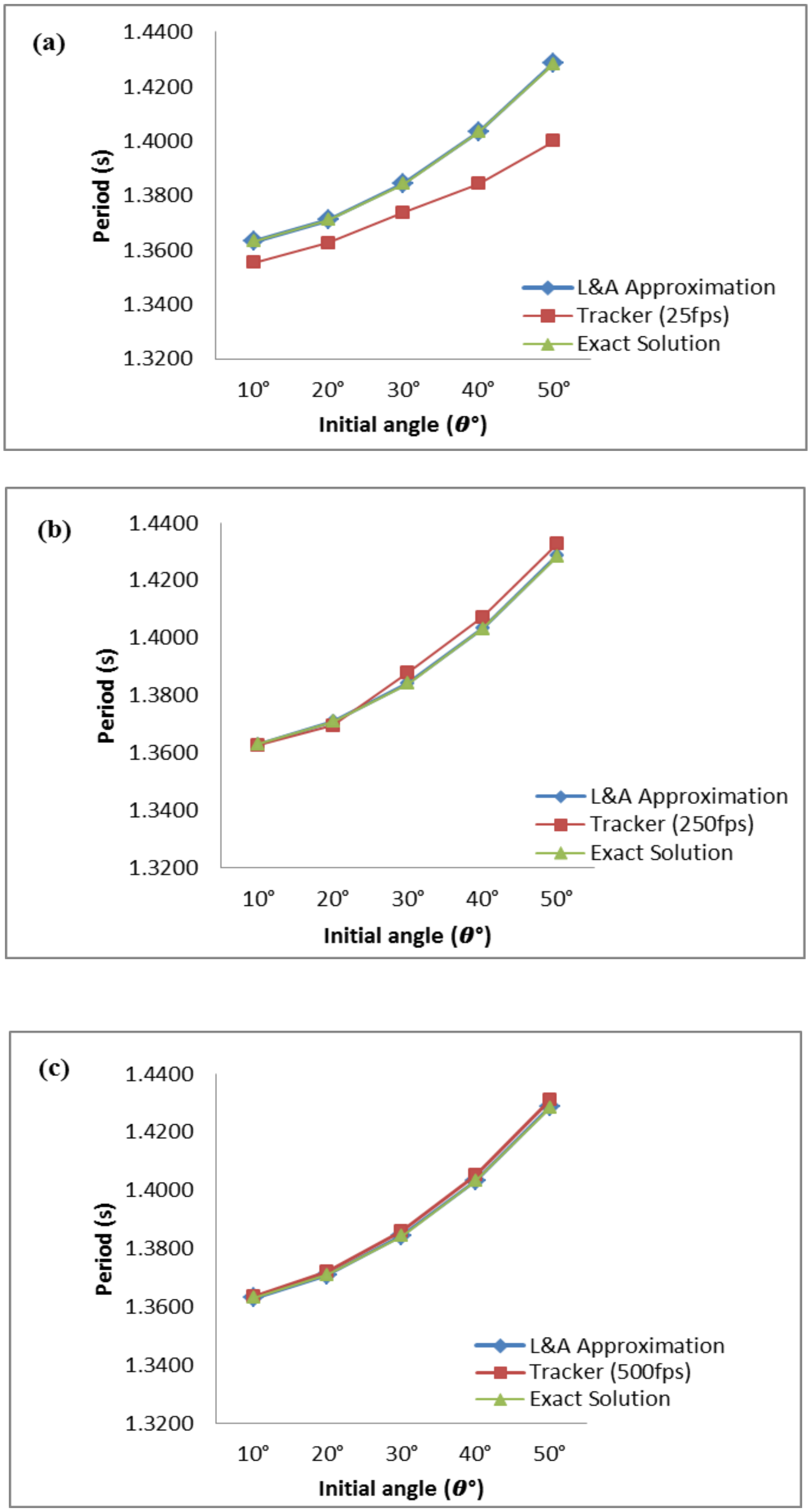

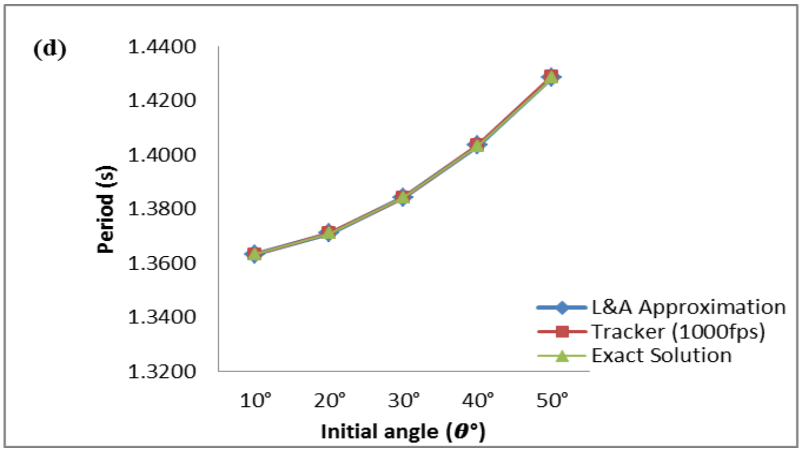

Figure 4: Comparison between Tracker video analysis method, L&A Approximation and exact period solution of period at different video frame rate (a) 25fps, (b) 250fps, (c) 500fps, (d) 1000fps

Table 1**:** Period of oscillation and relative errors between L&A approximation and our video analysis approach

Initial Angle ($\theta°$)	Exact period solution (s)	Lima and Arun Approximation (s)	Video Frame Rate							
			25 fps		250 fps		500 fps		1000 fps	
			Period (s)	% Error	Period (s)	% Error	Period (s)	% Error	Period (s)	% Error
10	1.3632	1.3632	1.3553	0.578	1.3627	0.038	1.3635	0.027	1.3632	0.005
20	1.3710	1.3710	1.3627	0.611	1.3695	0.112	1.3722	0.084	1.3710	0.003
30	1.3843	1.3843	1.3737	0.768	1.3879	0.263	1.3861	0.130	1.3843	0.003
40	1.4032	1.4033	1.3843	1.359	1.4072	0.276	1.4053	0.141	1.4034	0.007
50	1.4283	1.4286	1.4000	2.002	1.4326	0.277	1.4312	0.185	1.4290	0.026

CONCLUSION

The approach combining Tracker video analysis and high speed camera to study simple pendulum with large angle approximation has proven to be accurate when compared with the exact period solution and L&A approximation. This approach has shown that it can produce better results than using traditional method such as stop watch or expensive sensor based measuring equipment. Besides, it is also very interactive as students can analyse the motion of the pendulum in real time or slow motion from the video.

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